Pacing and Comparing

What Happens

This session is devoted to comparing the number of units that are needed to cover a particular distance with giant steps, baby steps, and a walking pace. Students get firsthand experience with the inverse relationship between the size of their steps and the number of steps needed to cover a certain distance. They also estimate the number of paces needed to reach a target, then check their estimates. Their work focuses on:

- using a nonstandard unit to measure distance
- comparing the effects of measurement using units of different sizes
- estimating distances
- collecting and analyzing data

Materials

- Index cards or stick-on notes

Ten-Minute Math: Estimation and Number Sense Once or twice during the next few days, use the activity Estimation and Number Sense. Remember, this short activity need not be done during math time.

Display an addition problem on the board for about a minute. For example:

25 + 10 + 25 or 31 + 46 + 3

During this minute, students mentally estimate the answer—no writing or calculator use allowed.

Cover the problem while students discuss their estimates.

Reveal the problem again and ask them to find a precise solution to the problem by using mental computation strategies.

The first two times you do this activity, use only addition problems. For variations, see pp. 75–76.

Activity

Pacing the Classroom

Recently we measured the length of our classroom in giant steps and baby steps. Taking giant steps or baby steps can be uncomfortable, though. A more widely used measurement is the *pace*, which is closer to a regular step. How can we measure someone's pace? Let's try with a volunteer.

Ask one student to take three or four regular walking steps, then freeze. Ask the class how long they think the room is in paces of that size. After they estimate the distance, have the same student pace off the length of the room while the class counts to keep track.

Working in the same pairs as in Session 1, the students estimate and count the number of paces it will take them to measure the length of the classroom. They can write their results on index cards or stick-on notes. You will record their data on a line plot, as before.

Let's record our results on a line plot. We need to decide on the beginning and end of the number line we'll use. Any ideas?

What can we see from this line plot? What can you say about the data?

Help students express their ideas as they describe the distribution of the data (see the **Teacher Note**, The Shape of the Data: Clumps, Bumps, and Holes, p. 13).

What do you see in the graph? Do you see any clusters of data? Did most of you get the same number of paces when you measured the length of the room? Is the range of data very wide? Are there any unusual values?

For an example of such a discussion, see the **Dialogue Box**, Describing the Shape of the Data (p. 12).

On another line plot, put the results of Session 1 (giant steps and baby steps) on the board and ask students to compare those data with their new results.

What can we say about our paces, our giant steps, and our baby steps? How do they compare?

This is likely to produce a discussion of the relationship between step size and number of steps. Students will argue with each other about this relationship and about reasons for the difference in results. They may have trouble articulating their ideas, and some will not be able to put their ideas into words. Students may assert that it takes more steps for bigger people; others will assert that it's fewer—again, the inverse relationship (the bigger the person the fewer the steps) may confuse some students. Some teachers who have worked with this unit believe that the fact that smaller numbers "win" here is unusual and important, so that students do not just look at large numbers without evaluating what they represent.

(See the **Dialogue Box**, Discovering Children's Beliefs About Numbers, p. 14, for another example of how classroom discussions can help you discover how your students think about number, measurement, and data.)

Questions like the following can produce fruitful discussion:

Which are the biggest steps? Did you take more baby steps or more giant steps? Why?

If I take 6 giant steps to walk along here, will I take about 30, or about 10, or more like 3 baby steps to cover the same distance?

Make sure that students give reasons for their statements.

Activity

Estimating How Far in Paces

Ask one student to stand in a fairly open part of the classroom. Select a target that is a moderate distance away ***in a straight line***. Have the student take three or four *paces* to help the others visualize the length of a pace.

You may want to dramatize the visualization process by "thinking aloud" your own way of making an estimate:

Let's see—I can see how long Chantelle's pace is, so I'll try to imagine: 2 ... 3 ... 4 ... 5 ... 6. About 6 paces to the desk, I think. Let's try it!

Now, how many paces is it from Chantelle to the globe?

Students estimate, the pacer paces the distance, and everyone counts. Repeat this two or three times, selecting different objects. You may want to use more than one student as a pacer.

Session 2 Follow-Up

Extension

Estimating Other Distances Students may develop some good strategies for estimating distances in paces. To practice these strategies, follow up with a whole-class activity. Select various points in the classroom and ask everyone to estimate the number of a specific student's paces between the points. Write down all estimates, then check by pacing to see how close the class estimates are. Visual estimation is an important component of measurement, and practice does make a difference!

D I A L O G U E   B O X

Describing the Shape of the Data

```
        X
        X  X
X  X    X  X  X
X  X    X  X  X        X              X
------------------------------------------
14 15  16 17 18 19 20 21 22 23 24
```

The students have just counted the number of paces it takes each of them to walk the length of the classroom. They have recorded their data on a line plot (above).

So what can you say about these pace data? Let's hear a few of your ideas.

Sean: Well, there are a lot at 16.

Chantelle: There's only one at 20 and at 23.

Kate: There are two each at 14 and 15.

What else do you notice?

Maya: Fourteen is the lowest.

Jeremy: Yeah. And 23 was the highest.

So the *range* was from 14 to 23. What else?

Annie: There's nothing at 19, 21, or 22.

Annie's noticing a lot of holes in this part of the data. Can anyone say any more about that?

Michael: Well, there's nothing at 12 or 13 either.

Yes, 14 is the lowest count and there's nothing below it. But this situation, that Annie noticed up here, is a little different. What can you say about that?

Su-Mei: Mostly, the paces go from 14 to 18, but sometimes you get something higher.

Can anyone add to that?

Sean: You must have really small paces if it takes you 23.

In fact, mathematicians have a name for a piece of data that is far away from all the rest. They call it an *outlier*. An outlier is an unusual piece of data—sometimes it might actually be an error, but sometimes it's just an unusual piece of data. It's usually interesting to try to find out more about an outlier. Who had the outlier in this case?

Ly Dinh: I did. And I counted twice, and Michael checked me, too, so I know it was 23 paces.

Jennifer: Maybe he's got smaller feet.

Any other theories about Ly Dinh's pace?

[*Later*] ... **Suppose someone asked you, then, "What's the typical number of paces to cross our classroom?" what would you say?**

Tamara: Well, I'd say 16.

[*Addressing the class as a whole, not just Tamara*] **Would 16 be a reasonable description of how many paces long our room is?**

Cesar: Yes, because most of us took 16 paces.

Any other ways to say this? Or any different ideas?

Midori: Well, I wouldn't say just 16.

Why not?

Midori: Well, there's really not that much difference between 16, 17, and 18. They're all really close together. I'd say 14 to 18, 'cause the 20 and 23 aren't what you'd usually get.

So Midori is saying she'd use an *interval* to describe the pace-length of our room, from 14 to 18, and Cesar said he'd say the length was about 16. What do other people think?

In this discussion, the class has moved gradually from describing individual features of the data to looking at the shape of the data as a whole. Throughout, the teacher tries to have students give reasons for their ideas and pushes them to think further by asking for additions or alternatives to ideas students have raised.

Moved from individual items of info to shape of Data

Notice how the teacher introduced, in context, the terms *interval*, *range*, and *outlier*. This is the best way to introduce such new vocabulary, when the ideas come up and the terms describe some particular real data.